Celeste Druce

Derivation of the Standard form of the equation for an ellipse

$\sqrt{(c-x)^2+y^2}+\sqrt{(c+x)^2+y^2}=R$

$\Rightarrow \sqrt{(c-x)^2+y^2}=R-\sqrt{(c+x)^2+y^2}$

$\Rightarrow (c-x)^2+y^2=R^2-2R\sqrt{(c+x)^2+y^2}+(c+x)^2+y^2$

$\Rightarrow c^2-2cx+x^2=R^2-2R\sqrt{(c+x)^2+y^2}+c^2+2cx+x^2$

$\Rightarrow 2R\sqrt{(c+x)^2+y^2}=R^2+4cx$

$\Rightarrow 4R^2((c+x)^2+y^2)=R^4+8R^2cx+16c^2x^2$

$\Rightarrow 4R^2(c^2+2cx+x^2+y^2)=R^4+8R^2cx+16c^2x^2$

$\Rightarrow 4R^2c^2+8R^2cx+4R^2x^2+4R^2y^2=R^4+8R^2cx+16c^2x^2$

$\Rightarrow 4R^2c^2+4R^2x^2+4R^2y^2=R^4+16c^2x^2$

$\Rightarrow 4R^2x^2-16c^2x^2+4R^2y^2=R^4-4R^2c^2$

$\Rightarrow (4R^2-16c^2)x^2+4R^2y^2=R^4-4R^2c^2$

$\Rightarrow \frac{4R^2-16c^2}{R^4-4R^2c^2}x^2+\frac{4R^2}{R^4-4R^2c^2}y^2=1$

$\Rightarrow \frac{4(R^2-4c^2)}{R^2(R^2-4c^2)}x^2+\frac{y^2}{\frac{1}{4}R^2-c^2}=1$

$\Rightarrow \frac{x^2}{\frac{1}{4}R^2}+\frac{y^2}{\frac{1}{4}R^2-c^2}=1$

$\Rightarrow a^2=\frac{1}{4}R^2$ and $b^2=\frac{1}{4}R^2-c^2$

$\Rightarrow a=\frac{1}{2}R$ and $b=\sqrt{\frac{1}{4}R^2-c^2}$